Default curves and the dynamics of credit spreads

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Introduction

In the investment grade corporate market, bonds generally trade on a spread basis to the Treasury curve or the swap curve, and the term structure of credit spreads tends to be upward sloping. High yield bonds and emerging market debt can behave quite differently; it is often, but not always, the case that:

- bonds with a low dollar price tend to trade on the basis of price, not yield; and
- when an issuer becomes distressed, its credit curve inverts.

These two phenomena are clearly related: for example, if there is a uniform drop in the dollar price of bonds with different maturities, this implies an inversion in the credit curve.

How can we model these phenomena? Broadly speaking, corporate bond spreads are determined by default risk, a liquidity premium, and a premium for investor risk aversion. For AAA rated securities, liquidity and risk aversion are clearly the main factors. However, for bonds rated BBB and below, it seems most reasonable to try to understand issuer-specific price or spread dynamics in terms of default risk. In fact, the two phenomena can both be explained by reference to the *term structure* of default risk – for instance, a rise in the near term risk of default will tend to have a near-uniform effect on the market values of bonds across maturities, so where near term default risk is dominant, bonds will tend to trade on a price basis.

It is important to note that the two phenomena noted above are rules of thumb, and the actual behavior of bond prices can be more complex. For example, Exhibit 1 shows that as the credit quality of Dillards deteriorated, its credit curve did indeed invert; however, when the bonds later rallied, the credit curve remained inverted. And Exhibit 2 shows that even as the perceived credit quality of Conseco deteriorated sharply, its credit curve remained positive and even steepened. This implies that the term structure of default risk can have rather complex dynamics.

We will show that this kind of complex behavior can emerge from a rather simple model. Before describing the model, we review some facts about marginal default probabilities and theoretical default spreads.[1]

Marginal default probabilities

An issuer's *marginal default probability* in year t is the probability that it will default in year t, conditional on its survival up to that year. Plotting the marginal default probability versus years forward gives you the issuer's *default curve*, which displays the term structure of default risk; it is analogous to a forward rate curve. It is obviously not possible to observe expected default curves directly (except perhaps via the credit derivatives market), but we can get a good impression of what default curves might look like by analyzing historical default rates.

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[1] For more details, see W. Phoa, *Advanced Fixed Income Analytics*, Fabozzi, 1998; and W. Phoa, *Credit ratings and yield spreads: the theoretical pricing of default risk*, Deutsche Morgan Grenfell (Sydney) research report, May 1994.

Exhibit 3 shows 'average' default curves for corporate issuers of various ratings, based on Moody's historical cumulative default rates. Observe that broadly speaking, investment grade corporates tend to have upward-sloping default curves; Bb-rated corporates tend to have mildly humped default curves; and B-rated corporates tend to have inverted default curves.

Given a default curve, and assuming a fixed recovery rate, it is possible to compute a *default spread* for a bond; this is the yield spread that makes the expected return on the bond equal to that on a bond with the same coupon and maturity, but no default risk.[2] For simplicity, all default spreads in this paper assume zero recovery. It is intuitively clear that the default spread is then just an average of the marginal default probabilities up to the bond maturity, weighted by appropriate discount factor: see Exhibit 4.

The goal is to find a model of default risk from which we can derive a default curve. Changes in the financial condition of an issuer should then correspond to changes in the model parameters, which result in shocks to the issuer's default curve and thus to (the term structure of) default spreads. That is, the model provides a link between fundamental credit analysis and theoretical changes in bond spreads and prices.

A model of default risk

Here is the simplest model from which some useful results can be derived. Note that though it is inspired by the standard option-theoretic view of capital structure, it is a 'real-world' model, not a pricing model in the sense of martingale pricing theory.[3]

A firm has a real market value, which is assumed to follow geometric Brownian motion. The firm's capital structure consists of debt and equity; and if the firm's total market value ever falls so far that the value of the equity becomes zero, the firm defaults. (This is clearly an oversimplification, since it implies 100% recovery on the bonds; however, a more realistic model yields qualitatively similar results, scaled to reflect a positive recovery rate. We will sacrifice consistency and stick with the simpler version for clarity.)

In order to run the model, we thus need to specify three parameters:

1. The volatility of total (real) firm value.[4]
2. The initial economic leverage, i.e. initial debt/market value of firm.
3. The planned leverage path, i.e. annual planned percent change in economic leverage.

For example, a firm which intends to gradually deleverage has a planned leverage path that reflects an annual decline in economic leverage.

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[2] This is not the same as the bond's risk-neutral spread in the context of martingale pricing theory; the relationship is described in P. Artzner and F. Delbaen (1995), "Default risk insurance and incomplete markets", *Mathematical Finance* **5**:187-195.

[3] A good survey of these more rigorous models is D. Lando, "Modelling bonds and derivatives with default risk", in M. A. H. Dempster and S. R. Pliska (eds.), *Mathematics of Derivative Securities*, Cambridge University Press, 1997.

[4] This is *not* the same as the equity volatility; it will be lower, since the equity volatility is leveraged.

It may be helpful to list, in a very stylized fashion, how these three parameters are related to credit events that may occur in the real world.

1. Volatility can be affected by a change in business conditions that creates greater uncertainty about future earnings, or by a change in management, or by actual or rumored M&A activity.

2. Initial economic leverage can be affected by a shock to the market value of the firm, e.g. a sudden decline in its equity price – which in turn can be caused by a variety of factors, such as a shock to earnings.

3. The planned leverage path can be affected by a change in dividend policy or planned asset sales, or by an unexpected increase in capex requirements.

In practice, a given credit event is likely to affect more than one of these parameters.

It should be noted that, since the model assumes that the value of the firm changes continuously, it does not take into account the possibility of sudden shocks to the firm's market value, e.g. the discovery of accounting fraud. However, modifying the stochastic process to permit such shocks would yield different, but qualitatively similar, results.

Simulation results

It is straightforward to implement the model and derive marginal default probabilities using Monte Carlo simulation. We have used baseline parameters which result in a default curve typical for a split-rated corporate issuer. The parameters are:

1. 20% volatility of real firm value.
2. 50% initial economic leverage.
3. Zero planned change in leverage.

The results are shown in Exhibits 5, 6 and 7. In each case, the graph shows the default curve under different scenarios (with the heavy solid line being the base case), while the table shows theoretical changes in bond spreads and bond prices under each scenario, assuming zero recovery in the event of default. In each case, results are based on 25,000 simulations; the derived default curves are slightly irregular due to numerical inaccuracy.

Overall, the interesting observation is that shocks to the three parameters cause quite different shifts in the default curve, and in each case positive and negative shocks also cause quite different kinds of shifts.

Exhibit 5 shows the impact of a change in economic leverage, e.g. due to an equity price shock. Note that a positive shock results in a roughly parallel fall in spreads (with a flattening at the short end), and a moderate negative shock results in a parallel rise in spreads (with a steepening at the very short end). However, a large negative shock results in a sharp inversion of the default curve and the spread curve, and a nearly uniform fall in bond prices across maturities.

Exhibit 6 shows the impact of a change in volatility, e.g. due to a change in management. A positive shock again results in a fall in spreads and a flattening in the spread curve, whereas a negative shock results in a 'humped' rise in spreads.

Exhibit 7 shows the impact of a change in planned leverage path, e.g. due to an unexpected increase in capex forecasts. A positive shock results in a nearly parallel fall in spreads, while a negative shock results in a rise in spreads (a sharp steepening at the short end, but nearly parallel beyond 4-5 years).

In practice, translating real world scenarios into parameter shifts is not straightforward: for example, estimating the volatility shift resulting from a change in management requires a fairly detailed historical analysis of analogous situations. And in the end, as with any valuation model, it is a question of judgment rather than science. But the point of analysis is that judgments be backed by evidence, and that their consequences be quantified rigorously.

Conclusions

We have used a rather simple and abstract model to derive some rather complex dynamics for the term structure of credit spreads. The model may provide a useful way to relate credit analysis to bond prices, by translating credit events into price scenarios. Potential applications include:

- Credit risk management: more sophisticated simulations of portfolio credit risk that take into account the varying impact of different kinds of credit event identified by analysts.

- Security selection (particularly in distressed situations): helping to assess value where a firm has issued bonds of varying maturities.

- Portfolio construction: assessing the term structure of default risk at a portfolio level, forecasting expected cashflows, and helping to ensure proper diversification.

In its current form, however, the model has a number of drawbacks: it is oversimplified, computationally intensive, and difficult to calibrate.

Also, no attempt has been made to adapt the model for use with emerging market debt. In the corporate case, capital structure gives a notion of default threshold; whereas for emerging markets, the analogs of "firm value" and "default threshold" are more elusive. Here, more than in the corporate case, default is partly voluntary, and the result of strategic interaction; one needs to specify a model consistent with the nature of this interaction.[5]

[5] See V. K. Aggarwal, *Debt Games*, Cambridge University Press, 1996. For an arbitrage-free model of corporate securities with credit risk, incorporating strategic interaction between equityholders and bondholders, see H. Fan and S. Sundaresan, *Debt valuation, renegotiations and optimal dividend policy,* PaineWebber Working Paper Series PW-99-05, Columbia Business School, March 1999.

Exhibits

Exhibit 1 ■ Spread history of selected Dillards issues

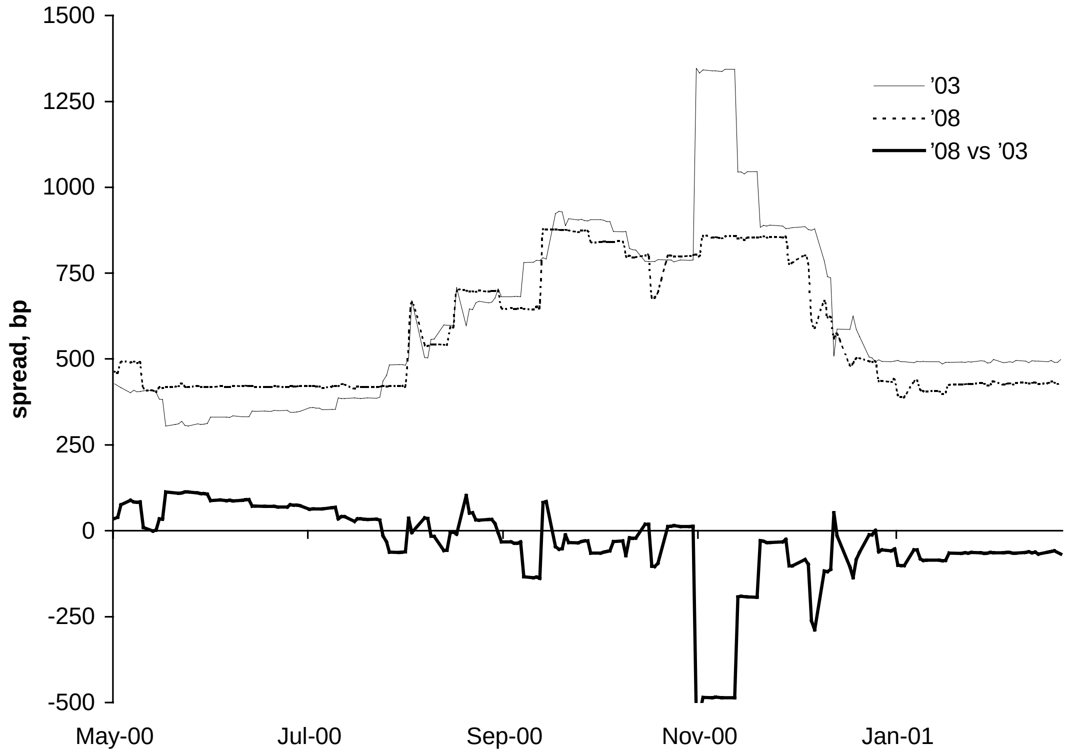


Exhibit 2 ■ Spread history of selected Conseco issues

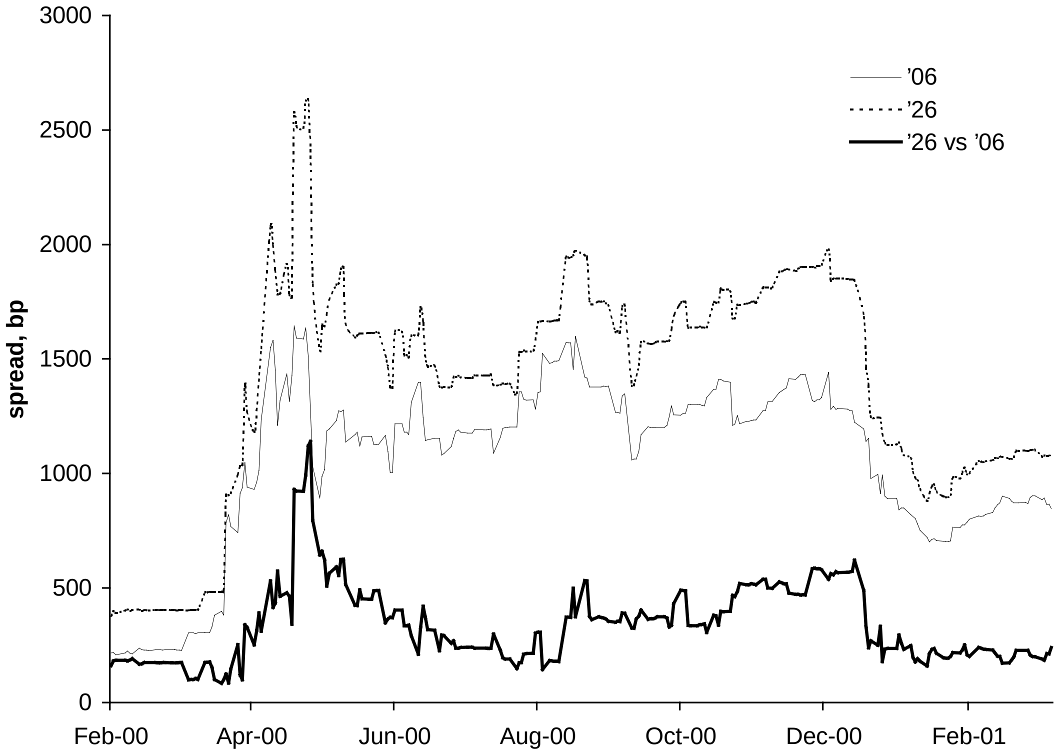


Exhibit 3 ■ Moody's derived marginal default probabilities

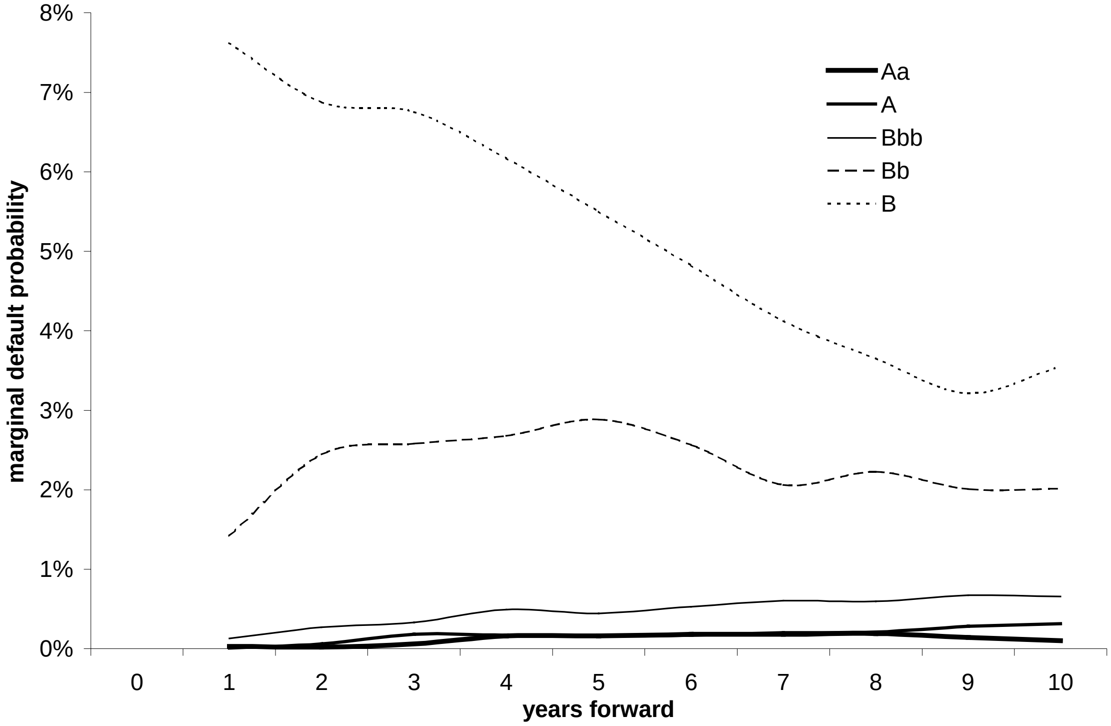
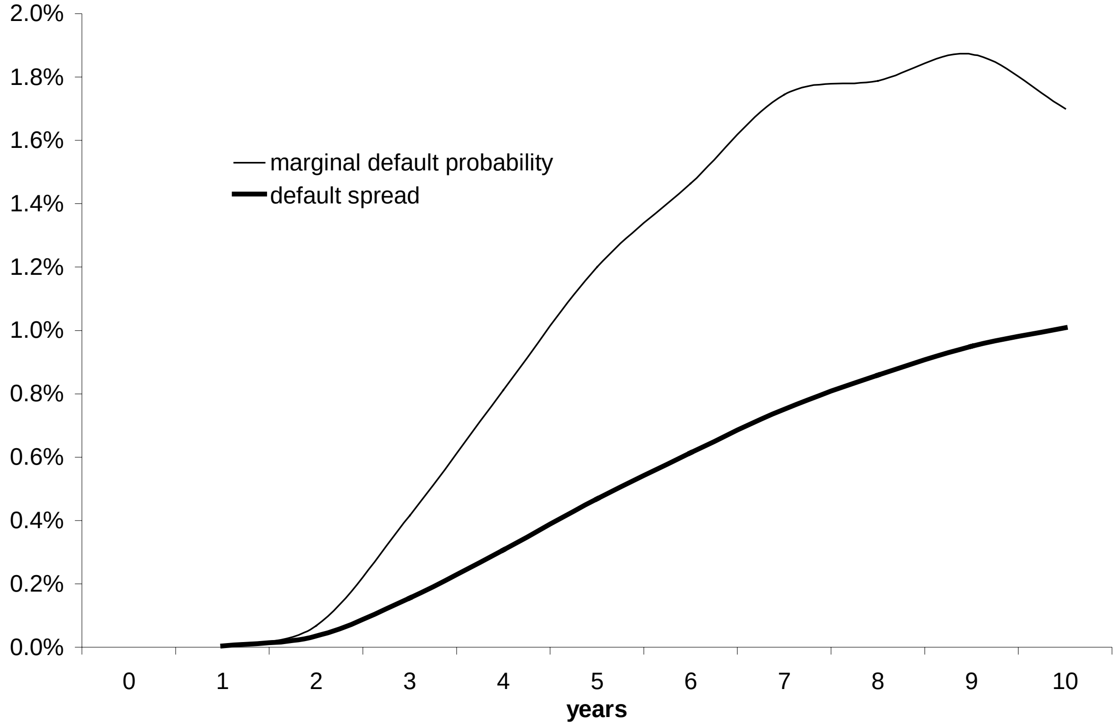


Exhibit 4 ■ Marginal default probability and (zero recovery) default spread



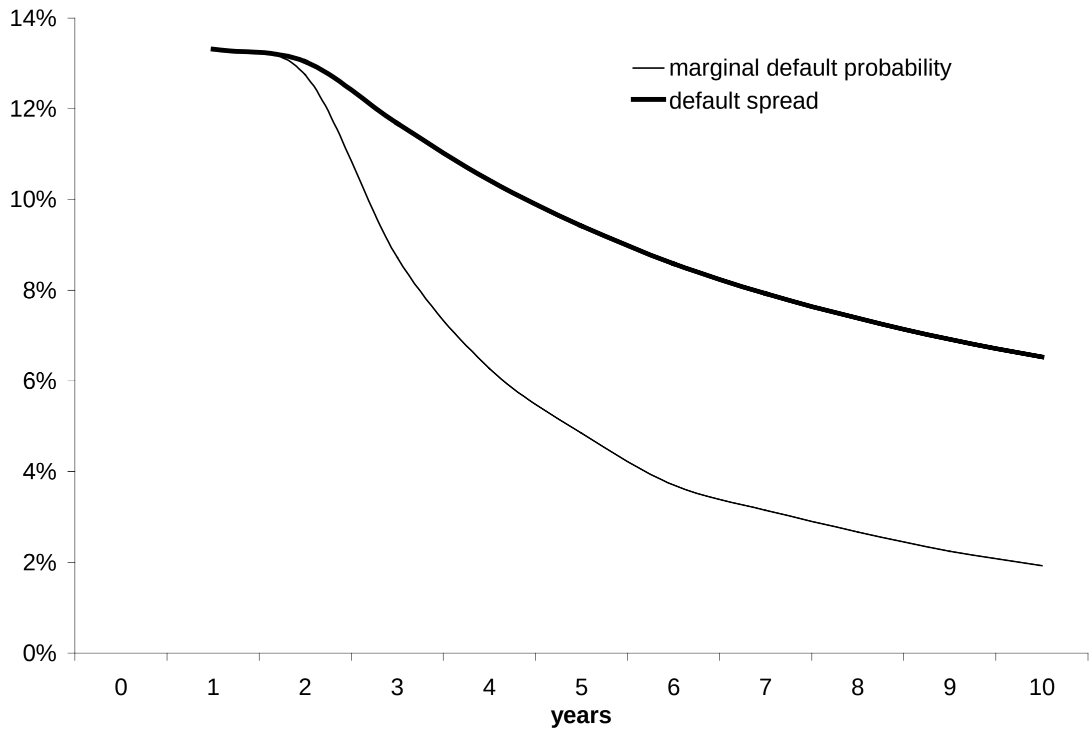
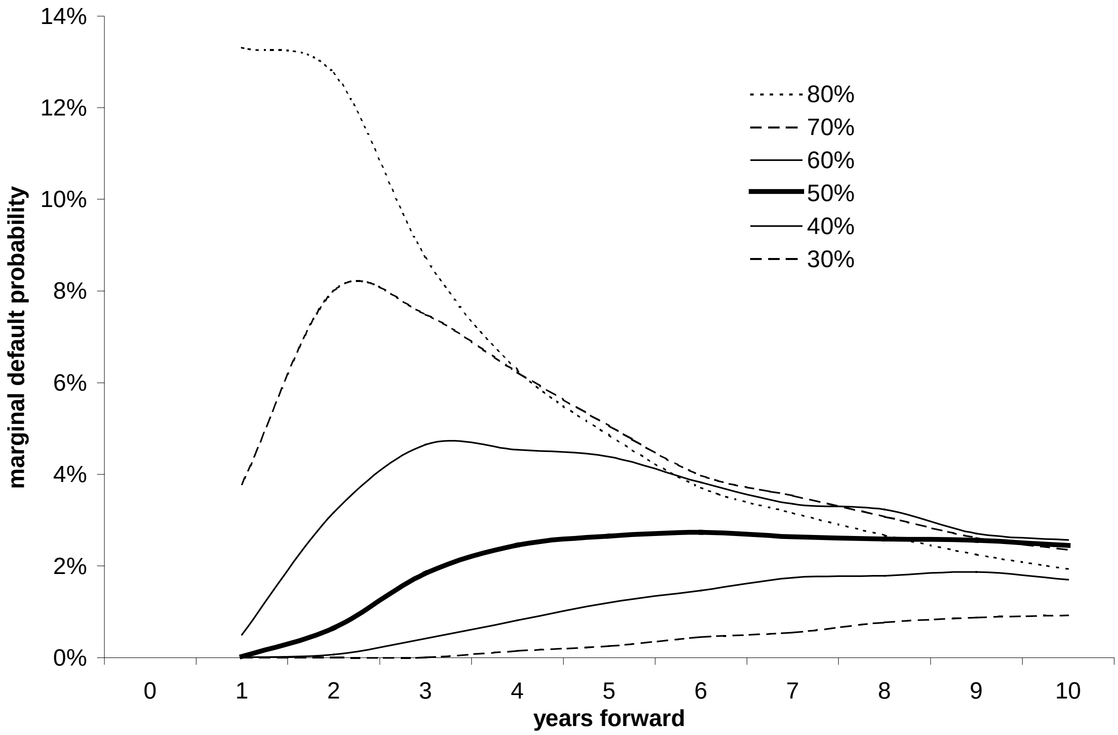


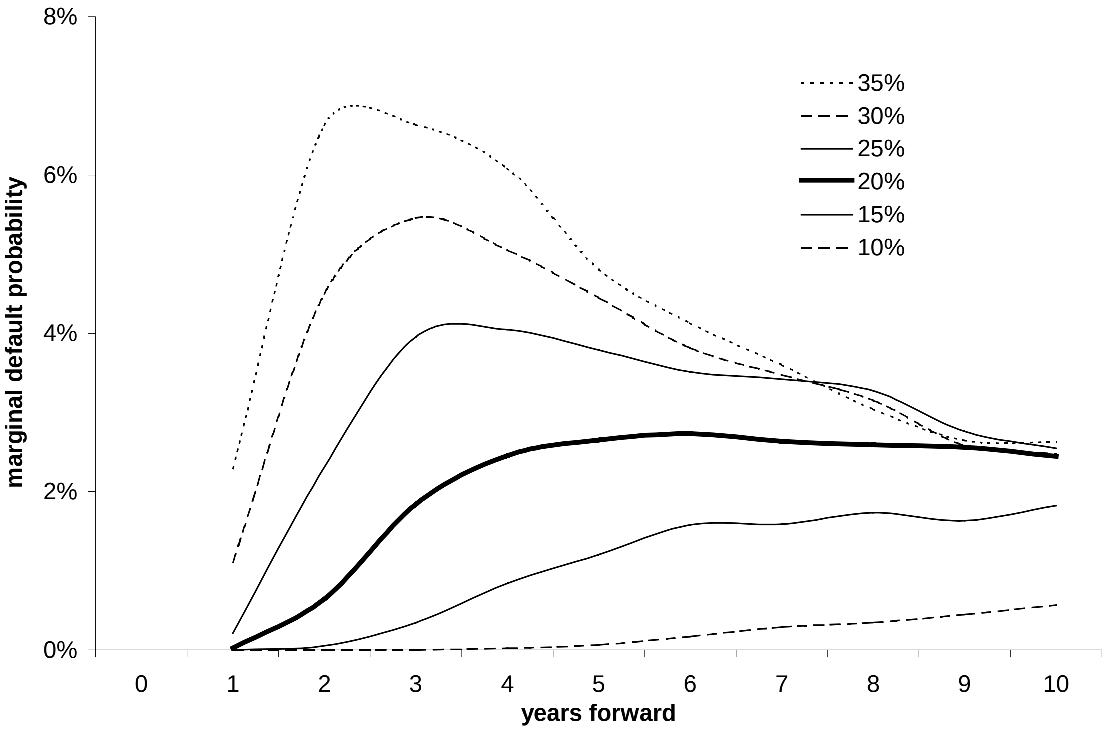
Exhibit 5 ■ Impact of change in initial economic leverage



Zero recovery assumption

	Change in yield (bp)						*Change in price*					
	10%	*15%*	*20%*	*25%*	*30%*	*35%*	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*
1	-2	-2	0	48	376	1329	0.02%	0.02%	0.00%	-0.47%	-3.71%	-13.13%
2	-33	-29	0	147	551	1271	0.63%	0.56%	0.00%	-2.84%	-10.63%	-24.51%
3	-80	-65	0	190	555	1087	2.27%	1.83%	0.00%	-5.35%	-15.66%	-30.65%
4	-115	-88	0	194	514	924	4.23%	3.23%	0.00%	-7.12%	-18.88%	-33.93%
5	-138	-98	0	190	465	797	6.17%	4.40%	0.00%	-8.52%	-20.84%	-35.74%
6	-151	-102	0	178	415	695	7.93%	5.38%	0.00%	-9.37%	-21.82%	-36.53%
7	-158	-101	0	165	376	617	9.46%	6.03%	0.00%	-9.90%	-22.48%	-36.92%
8	-161	-99	0	155	342	554	10.73%	6.60%	0.00%	-10.35%	-22.84%	-37.01%
9	-161	-96	0	142	311	501	11.86%	7.06%	0.00%	-10.46%	-22.89%	-36.84%
10	-161	-94	0	132	286	458	12.82%	7.53%	0.00%	-10.55%	-22.85%	-36.55%

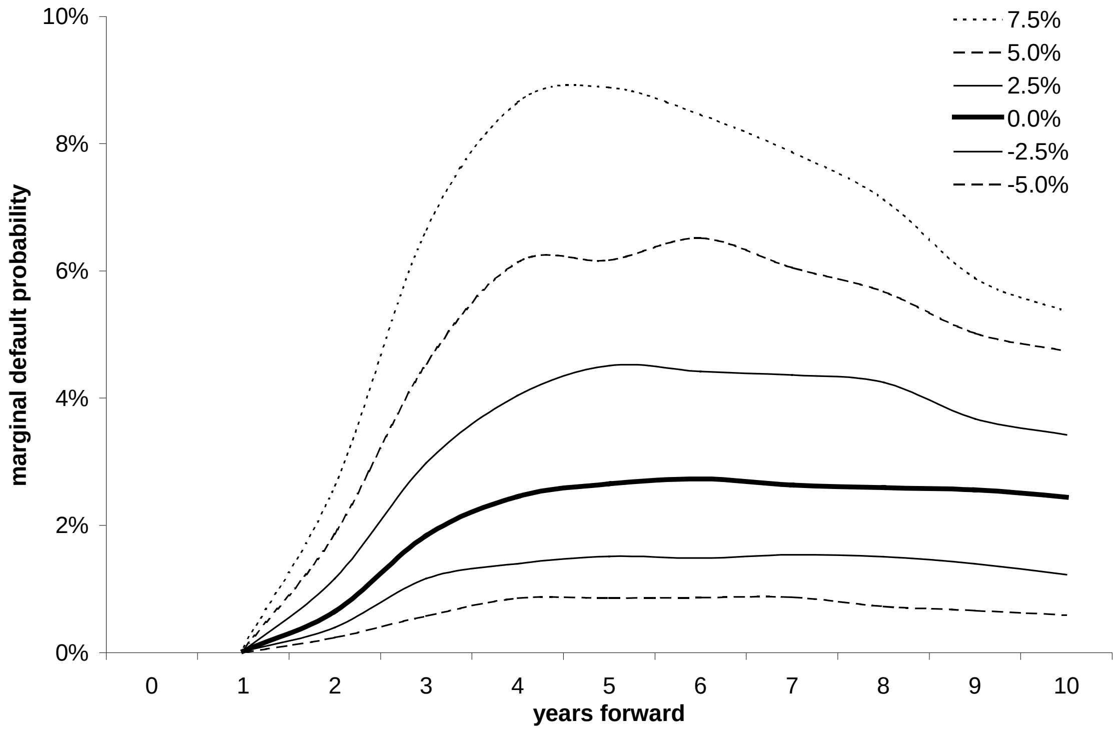
Exhibit 6 ■ Impact of change in volatility



Zero recovery assumption

	Change in yield (bp)						*Change in price*					
	10%	*15%*	*20%*	*25%*	*30%*	*35%*	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*
1	-2	-2	0	19	108	227	0.02%	0.02%	0.00%	-0.19%	-1.07%	-2.24%
2	-33	-30	0	91	243	408	0.63%	0.58%	0.00%	-1.75%	-4.69%	-7.87%
3	-81	-68	0	129	281	431	2.27%	1.91%	0.00%	-3.64%	-7.92%	-12.15%
4	-118	-90	0	136	276	415	4.34%	3.29%	0.00%	-4.99%	-10.13%	-15.24%
5	-144	-99	0	132	259	379	6.43%	4.46%	0.00%	-5.91%	-11.59%	-17.00%
6	-160	-102	0	124	237	344	8.41%	5.35%	0.00%	-6.51%	-12.43%	-18.08%
7	-169	-102	0	118	218	314	10.12%	6.12%	0.00%	-7.09%	-13.05%	-18.80%
8	-175	-100	0	113	201	286	11.69%	6.72%	0.00%	-7.57%	-13.45%	-19.12%
9	-178	-100	0	105	183	261	13.09%	7.34%	0.00%	-7.71%	-13.48%	-19.20%
10	-179	-97	0	97	169	242	14.28%	7.73%	0.00%	-7.78%	-13.51%	-19.33%

Exhibit 7 ■ Impact of change in planned leverage path



Zero recovery assumption

	Change in yield (bp)						*Change in price*					
	10%	*15%*	*20%*	*25%*	*30%*	*35%*	*10%*	*15%*	*20%*	*25%*	*30%*	*35%*
1	-159	-86	0	87	151	216	1.57%	0.85%	0.00%	-0.86%	-1.49%	-2.13%
2	-148	-80	0	83	148	214	2.85%	1.55%	0.00%	-1.61%	-2.85%	-4.13%
3	-146	-79	0	86	158	236	4.12%	2.23%	0.00%	-2.42%	-4.46%	-6.67%
4	-147	-81	0	91	173	264	5.41%	2.98%	0.00%	-3.35%	-6.37%	-9.71%
5	-149	-83	0	97	185	288	6.69%	3.73%	0.00%	-4.36%	-8.29%	-12.89%
6	-152	-86	0	101	196	304	7.96%	4.50%	0.00%	-5.33%	-10.31%	-15.98%
7	-153	-87	0	105	204	316	9.15%	5.21%	0.00%	-6.30%	-12.20%	-18.89%
8	-154	-88	0	108	209	322	10.32%	5.88%	0.00%	-7.22%	-13.96%	-21.54%
9	-156	-89	0	108	210	323	11.46%	6.56%	0.00%	-7.96%	-15.46%	-23.71%
10	-157	-90	0	108	211	321	12.55%	7.22%	0.00%	-8.61%	-16.86%	-25.66%