Implications of Merton models for corporate bond investors

Wesley Phoa

Capital Strategy Research
11100 Santa Monica Boulevard
Los Angeles, CA 90025

Introduction and history

In the past couple of years, participants in US capital markets have observed a strong link between equity markets and corporate bond spreads:

- When stock prices fall, bond spreads tend to widen; but
- The relationship seems nonlinear: it appears strongest when stock prices are low.

A typical example is shown in Exhibit 1, which plots the relationship between the daily stock price of Nextel and the daily spread on its cash pay bonds. When the stock price was over $20, the relationship appeared weak or absent; but when the stock price fluctuated below that level, there was a very strong link.

Have equity and bond markets always behaved like this, or is the recent period anomalous? Exhibit 2a plots generic spreads on single-A and single-B rated corporates against the (log of the) US equity index level, using data from the past ten years. In the period since 1998 there is a strong relationship; but in the 1992-1998 period the relationship seems weaker, if it exists at all. To repeat the question: is the period since 1998 unusual?

Exhibit 2b plots US corporate bond spreads against the US equity index level during the 1919-1943 period. The resemblance to Exhibit 1 is striking – there is a clear relationship, which was stronger when equity prices were lower. (Note that spreads on low quality bonds were tighter in 1921 than they were when the equity index revisited comparable levels after the Crash, presumably because firms accumulated more debt in the intervening period.)

Exhibit 2c plots spreads on US railroad bonds against the US railroad stock index during the 1857-1929 period. (UK gilts are used as a risk-free yield benchmark, consistent with market practice during that period; note that there is no currency component to the spread, since both countries were on the gold standard for almost the whole period.) Exactly the same relationship appears in this graph. Furthermore, deviations from this relationship mostly have reasonable explanations; for example, spreads were unusually wide in the late 1860s/early 1870s, but this was a period when railroads' capital structures were being dishonestly manipulated on a massive scale.

It therefore seems that this link between the equity market and the corporate bond market has always existed. But it was only in the 1970s that a theoretical framework was developed, within which formal models of the relationship could be constructed.

Model overview, uses and caveats

Merton proposed in 1974 that the capital structure of a firm can be analyzed using contingent claims theory: debtholders can be regarded as having sold a put option on the market value of the firm; and equityholders' claim on the firm's value, net of its debt obligations, resembles a call option. In this framework, the meaning of default is that the value of the firm falls to a sufficiently low level that "the put option is exercised" by liquidating the firm or restructuring its debt.

This idea led to the so-called "structural models" of credit risk, which assume that (a) default occurs when the market value of the firm falls below a clearly defined threshold, determined by the size of the firm's debt obligations, and (b) the market value of the firm can be modeled as a random process in a mathematically precise sense. Taken together, these assumptions make it possible to calculate an estimated default probability for the firm. Note that the precise estimate will depend on the assumed default threshold, the nature and parameters of the random process used to model the firm's value, and possibly other technical details; different choices lead to different models.[1]

However, all the different structural models have a strong family resemblance. First, they have similar inputs; the key inputs tend to be:

1. The capital structure of the firm.
2. The market value of the firm, usually derived from its stock price.
3. The volatility of the firm's market value, usually derived from stock price volatility.

Second, they make qualitatively similar predictions. In particular, they all imply that:

- The credit risk of a firm rises as its stock price falls; but,
- This relationship is nonlinear, and is most apparent when the stock price is fairly low.

And this is precisely the pattern observed in Exhibits 1, 2a, 2b and 2c.

Since the late 1990s there has been a dramatic rise in the popularity of structural models. KMV Corporation pioneered the approach and built a formidable global client base among commercial banks, but other models have recently gained substantial followings among other capital market participants; these include CSFB's CUSP Model, RiskMetrics' CreditGrades and Bank of America's COAS. The Capital Group has developed a proprietary model along similar lines.

This article discusses whether structural models should be of interest to corporate bond investors. It assumes some general familiarity with the theoretical details.

In gauging the reliability and usefulness of structural models of credit risk, it is important to understand that they can be used in a number of different ways: for example,

1. To estimate default risk.
2. To predict rating transitions (especially downgrades).
3. To identify relative value opportunities within a specific firm's capital structure.
4. To predict changes in corporate bond spreads.
5. To identify relative value opportunities within the corporate bond market.
6. To assess the sensitivity of corporate bond spreads to equity prices.

[1] For an overview of structural credit risk models, as well as the alternative "reduced-form" approach commonly used to price credit derivatives, see K. Giesecke, *Credit risk modeling and valuation: an introduction*, Humboldt-Universität zu Berlin, August 19, 2002. For a survey of empirical results, see Y. H. Eom, J. Helwege & J.-Z. Huang, *Structural models of corporate bond pricing: an empirical analysis*, EFA 2002 Berlin, February 8, 2002.

It may turn out that a model performs well in some of these applications, but is useless for others. And it is important to understand that in each case, the key premises differ. Every proposed application of a model assumes that:

1. The assumptions underlying the model are reasonably accurate.

However, all applications except the first make further strong assumptions about the way in which different market participants process new information relevant to credit risk. Remembering that the key input to these models is the stock price, the corresponding assumptions are:

2. Rating agencies sometimes lag equity markets.
3. Equity and bond markets sometimes process information in inconsistent ways.
4. Bond markets sometimes lag equity markets.
5. Bond markets sometimes process information less efficiently than equity markets.
6. Equity and bond markets eventually process information in consistent ways.

Assumptions 1 and 6, and perhaps 2, are quite plausible; assumptions 3, 4 and 5 are more questionable. Research within the Capital Group has been mainly concerned with the last application: assessing the sensitivity of corporate bond spreads to equity prices. It is therefore assumptions 1 and 6 that play the most important role.

To see why assessing equity sensitivity is important, it is helpful to adopt the perspective of asset allocation. One reason to own bonds is that they provide diversification versus equity returns: bond investments should hold up well in periods where equities have poor returns. Therefore, it is not rational to invest an excessive amount in corporate bonds which have a high correlation with equities. However, structural models imply, and experience shows, that this correlation varies with equity prices. Therefore, a prudent approach to corporate bond investment in a portfolio context should take equity prices into account. This is the place where structural models can play a crucial role in credit risk management.

A final important use for structural models is the estimation of default correlations (or joint default probabilities), which are crucial in applications such as portfolio credit risk aggregation and CDO modeling. Default correlations are hard to measure, particularly for the investment grade universe: for example, default data is far too sparse; both default and rating transition are hard to use directly because of timing problems; and the use of bond spread data tends to understate correlations. The Merton approach suggests that default correlations may be inferred from directly observable equity market data. Note that default correlations need not be equal to equity (or firm value) correlations; nor can one estimate the default correlation just by measuring the correlation of changes in the estimated default probability. However, the calculations do turn out to be computationally quite tractable.[2]

[2] For a closed form formula, see C. Zhou, *Default correlation: an analytical result,* Finance and Economics Discussion Series 1997-27, Board of Governors of the Federal Reserve System, May 1, 1997.

Some empirical results

Since the technical details of structural models are covered elsewhere, it seems more helpful to organize the discussion around some empirical illustrations. To begin with, Exhibits 3a to 3e indicate how the model works, using the example of Nextel Communications.

Exhibit 3a shows the historical stock price and "distance to default". The latter is derived from Nextel's enterprise value (determined by its stock price) and the amount of debt in its capital structure; a distinction is made between long and short term debt. Note that Nextel's leverage increased during this period, so the fact that the stock price was the same on two different dates does not imply that the distance to default was the same.

Exhibit 3b shows the estimated historical volatility of Nextel's enterprise value; this estimate has fluctuated between 35% and 60%, as equity volatility has varied, so it would clearly not be valid to use a constant volatility input. Note that this volatility is not directly observable, and different models estimate it in different ways. Most models derive it from equity volatility: for example, KMV uses historical equity volatilities computed using a fixed window; the Capital Group's model uses historical volatilities computed using a simple exponentially weighted scheme; and CUSP uses option implied volatilities if they are available, and a GARCH estimate if they are not. There are a few models (such as COAS) do not look at equity volatility, but use the volatility implied by the market price of debt instead; this is an interesting alternative, though it is impracticable if the bonds are illiquid and/or there is a substantial amount of bank debt in the capital structure.

Exhibit 3c shows the historical daily stock price and credit risk measure, labeled "bond risk" on the graph. This is the probability that Nextel's enterprise value will cross the default threshold within the next 12 months; however, since default need not be an automatic event, this risk measure should be interpreted as a "probability of distress", or the probability of a severe financial crisis, rather than a literal default probability.

Exhibit 3d shows the historical daily credit risk measure (probability of distress) and the historical daily spread on a specific Nextel cash pay bond. As expected, the bonds tend to widen when the probability of distress rises, and vice versa; though it does seem that in 2000-2001 the bond market was somewhat slow to respond to an increase in risk.

Exhibit 3e is a scatter plot of the bond spread against the probability of distress. In theory, this should be an upward sloping line or curve, and the observations do show this pattern. That is, the model "works". However, there are two other interesting phenomena.

1. The curve slopes upward more sharply at high levels of risk. This is observed for many high yield issuers, and may reflect declining recovery value assumptions.
2. In the more recent period, the curve as a whole has shifted upward. This is not a widespread phenomenon, and may reflect an increased level of investor risk aversion towards the high yield wireless sector in 2002, independent of current equity prices.

Some additional findings emerge when looking at further examples, this time drawn from the investment grade universe.

Exhibit 4a shows, for Ford, the historical probability of distress and the historical bond spread; as before, the bonds tend to widen when the probability of distress rises, and vice versa. Rating actions are also marked on the graph: the larger white circles indicate Moody downgrades and the smaller circles mark dates when Ford was put on negative watch; gray circles mark more recent rating actions by S&P. In some cases the bonds seemed to widen in response, but in some cases the bonds had clearly widened in anticipation, and often the performance of the bonds was not related at all to a rating action. This example shows that for bond investors, predicting rating transitions is not the most important application.

Exhibit 4b is a scatter plot of the bond spread against the probability of distress. In this case the observations cluster nicely around an upward sloping straight line. There is an excellent relationship between the actual spread on the bonds and the model's estimate of credit risk. Thus, the model provides a good way to estimate the equity sensitivity of Ford bonds.

Exhibit 4c shows a different way of visualizing how this sensitivity has changed over time. The thick solid line shows Ford's historical stock price. The thin dotted line marks the "critical range" where the equity sensitivity of the bonds rises significantly, while the thin solid line marks the point of maximum sensitivity. (Note that if both the capital structure and volatility were constant over time, these lines would be horizontal.)

Exhibit 5a shows, for Sprint, a scatter plot of the bond spread against the probability of distress. Again, the observations cluster nicely around an upward sloping straight line, indicating that the model is very consistent with the market behavior of the debt; note that there is more scatter at higher levels of risk. Does scatter represent trading opportunities?

Exhibit 5b shows the daily probability of distress, the daily bond spread, and the daily credit default swap spread (plus the 5-year swap spread). An analysis of this data indicates that none of these three markets consistently leads the other two. The bond spread and CDS spread are tightly linked; one market occasionally lags the other, but only by a day. The probability of distress (which reflects the equity market) is much less tightly coupled; it sometimes leads the bond and CDS markets by a week or more – and is hence a trading signal – but it also sometimes lags.[3]

Structural models will only give useful trading signals to a bond investor when bond markets are less efficient than equity markets (perhaps due to the constraints affecting bond investors, and/or their bounded rationality). An example might be when a credit event affects an industry as a whole, causing bond investors to rapidly reduce their industry allocation by selling across the board; this may trigger a uniform widening in bonds across different issuers, even though the actual rise in credit risk may vary from firm to firm.

[3] Note that if there are cash instruments trading at a significant discount to par, both an implied default probability and an implied recovery rate (for bonds) can be computed from the observed credit default swap basis. Furthermore, a structural model can be used to estimate an "expected recovery rate" based on the conditional mean exceedance (i.e. the mathematical expectation of the firm value conditional on its dropping below the default threshold). A would-be arbitrageur might attempt to profit from discrepancies between the default probabilities and recovery rates implied by the equity and CDS markets; unfortunately, neither estimate of the recovery rate is very robust. A further problem is that priority of claims is often violated in the event of default and restructuring; this complicates the analysis of specific debt securities.

Finally, the above analysis assumes a constant capital structure going forward. Investors with longer term forecasting horizons might find it useful to extend the model by incorporating discretionary capital structure choices triggered by exogenous factors, such as macroeconomic conditions.[4]

Feedback loops and capital structure dynamics

As the popularity of structural credit risk models has grown – and particularly since Moody's acquired KMV – some market participants have suggested that the widespread use of these models creates excess volatility in debt markets and even leads to liquidity crises. The underlying complaint is that models which refer to stock prices are in some sense circular.

This concern is most commonly expressed at the level of the individual firm. The use of the KMV model by commercial banks can lead to an unfortunate feedback loop affecting highly levered companies: when the stock price falls, the model says that the credit risk of the company has risen; this causes banks to restrict access to credit, and also raises financing costs by pushing up bond spreads; this in turn puts financial pressure on the company, leading to a further decline in the stock price. Of course, this can only occur if the firm has an insufficiently liquid balance sheet. During 2002, several high profile firms in the telecommunications and energy sectors are said to have been affected.

It is hard to model this effect at the firm level, but stylized numerical simulations at the industry level provide some interesting results. Consider the following ideal industry model:

- The industry begins with a fixed allocation of debt and equity.
- Banks provide all the debt, and in doing so maintain fixed capital ratios.
- Banks fund themselves at a constant rate (e.g. a fixed spread over a risk-free rate).
- The industry receives a growing revenue stream which is used to service debt.
- Revenues in excess of debt service increase the industry's equity base.
- Individual issuer defaults occur at a rate predicted by a structural credit risk model.
- The stream of debt service payments causes bank capital to rise.
- Individual issuer defaults trigger loan losses, causing bank capital to fall.
- A fixed percentage of any increase in bank capital is allocated back to the industry.

Exhibits 6a and 6b compare the impact of two different loan pricing policies, static and risk based. More precisely, the two policies are:

1. New loans priced at the same rate as the original debt.
2. New loans continually re-priced with reference to the structural credit risk model.

(Risk-based pricing is assumed to use a spread based on the model's estimated default probability times a fixed expected loss rate; this is broadly consistent with the policy implied

[4] See, e.g., Korajczyk, R. & Levy, A., "Capital structure choice: macroeconomic conditions and financial constraints", *Journal of Financial Economics* **68** (April 2003).

by the forthcoming Basel II regime. It is also assumed that the model is reliable, i.e. the actual default rate is equal to the model's estimated default probability.)

In each case the scatter plot compares the returns to industry equity investors and the return on that portion of bank tier one capital allocated to the industry. The series of points shows how these returns evolve over time (the arrows indicate the direction of time). The model parameters are calibrated so that initial returns to both industry equity investors and bank tier one capital are quite attractive.

Exhibit 6a assumes static loan pricing. In this simulation, returns to both industry equity investors and bank tier one capital remain high for a while. Then, as leverage rises, there is an increase in the industry default rate. Return on bank capital falls sharply because of credit losses. However, as banks continue to extend loans at the same interest rate, equity returns for industry investors remain high; the benefit of leverage for equity holders offsets the damage done by defaults. Finally an equilibrium is reached, in which firms are more highly levered than initially, industry equity returns are higher, but banks suffer constant negative returns on tier one capital due to the higher equilibrium rate of defaults.

Exhibit 6b assumes risk based loan pricing. In this simulation, returns to both industry equity investors and bank tier one capital again remain high for a while. Then, as before, as leverage rises, there is an increase in the industry default rate. Banks adjust loan pricing, but return on bank capital still falls because there is always an existing stock of debt whose interest rate is too low. Meanwhile, return on industry equity capital falls since debt service eats up a higher and higher proportion of revenue; this causes leverage to spiral even higher. Eventually debt service exceeds revenue. There is no equilibrium. Instead, returns on both industry equity and bank capital become increasingly negative until both are wiped out.

These findings should not be taken too literally. The premises are not realistic: for example, it is implausible that banks would be willing to absorb credit losses forever, and it is implausible that the industry would never raise new equity in the capital markets (although this may become extremely difficult in periods of distress). So it is not yet possible to make quantitative real world predictions using this approach. The qualitative results remain a nagging worry rather than a concrete forecast.[5]

Structural models of credit risk can be powerful risk management tools, and perhaps even useful trading tools. Although they only became popular rather recently, they do seem to reflect timeless relationships between the corporate bond and equity markets. However, as the KMV model and its competitors exert an increasing influence on banks' credit decisions, investor behavior and possibly even rating agency actions, it is legitimate to ask whether there is a tradeoff between capital market efficiency and market stability.[6]

[5] It is surprisingly difficult to devise more realistic models of capital structure dynamics. Even sophisticated models have trouble accounting for the mix of debt and equity observed in the real world; for example, see M. Dewatripont & P. Legros, *Moral hazard and capital structure dynamics*, CARESS Working Paper 02-07, July 5, 2002.

[6] The author would like to thank Eknath Belbase and Ellen Carr for useful comments.

Exhibits

Exhibit 1 ■ Nextel: bond spread vs. stock price

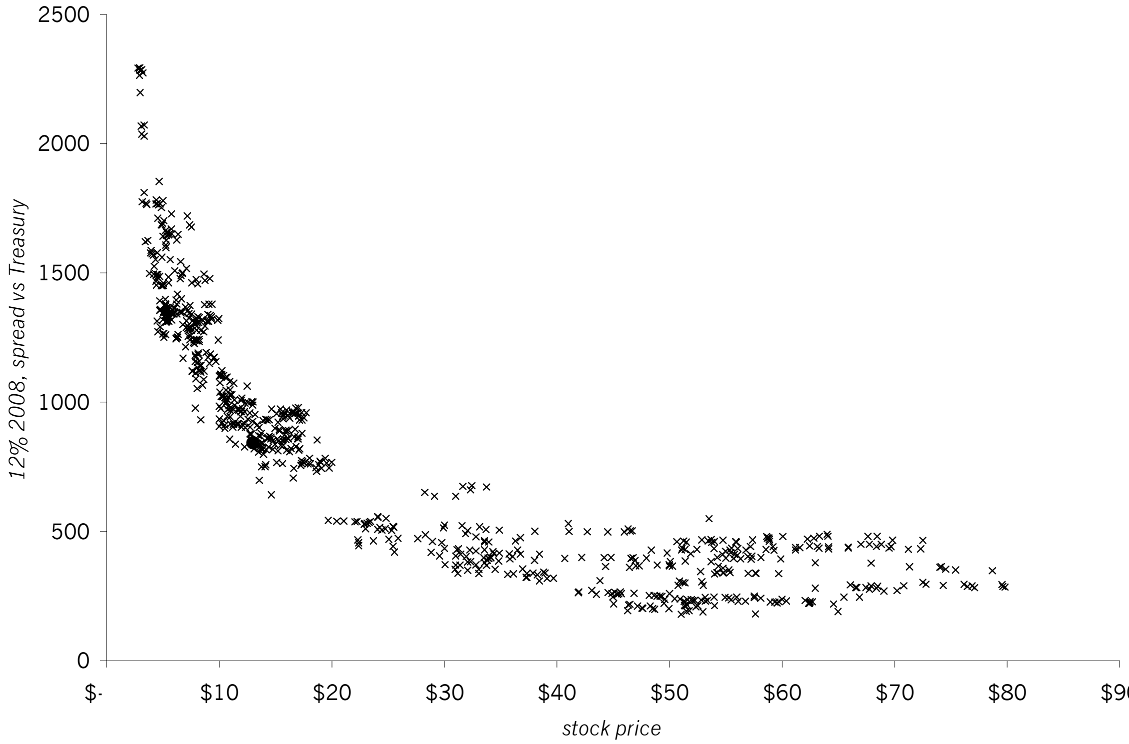

Exhibit 2a ■ US equity market and corporate bond spreads, 1992-2002

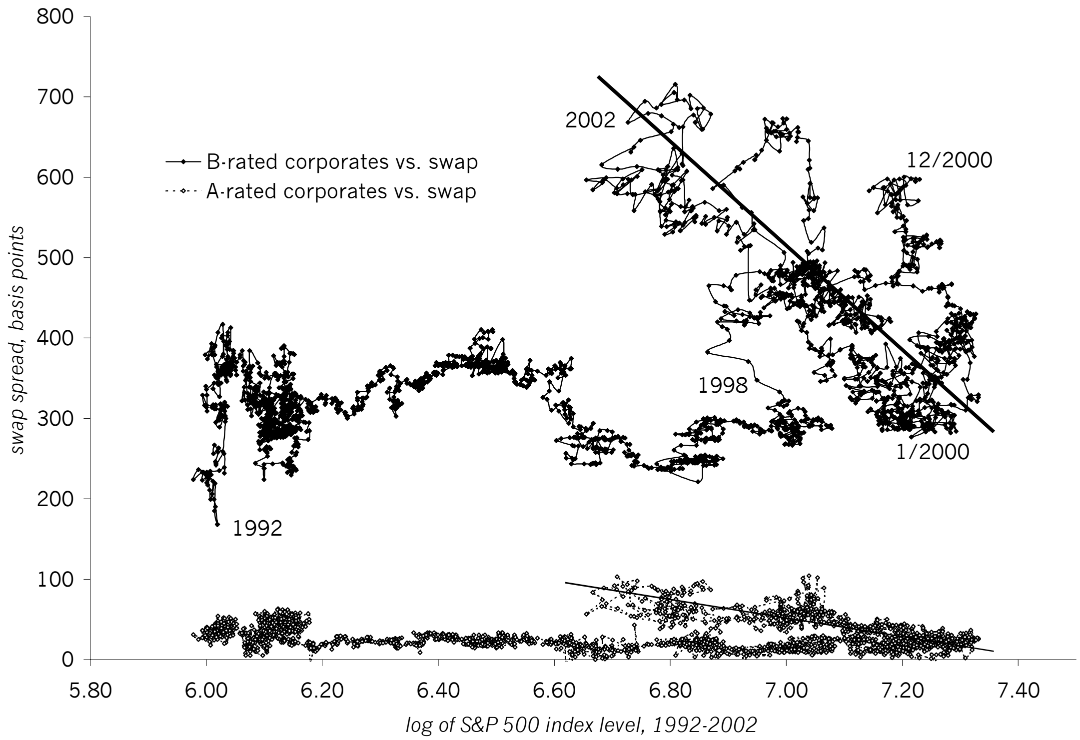

Exhibit 2b ■ US equity market and corporate bond spreads, 1919-1943

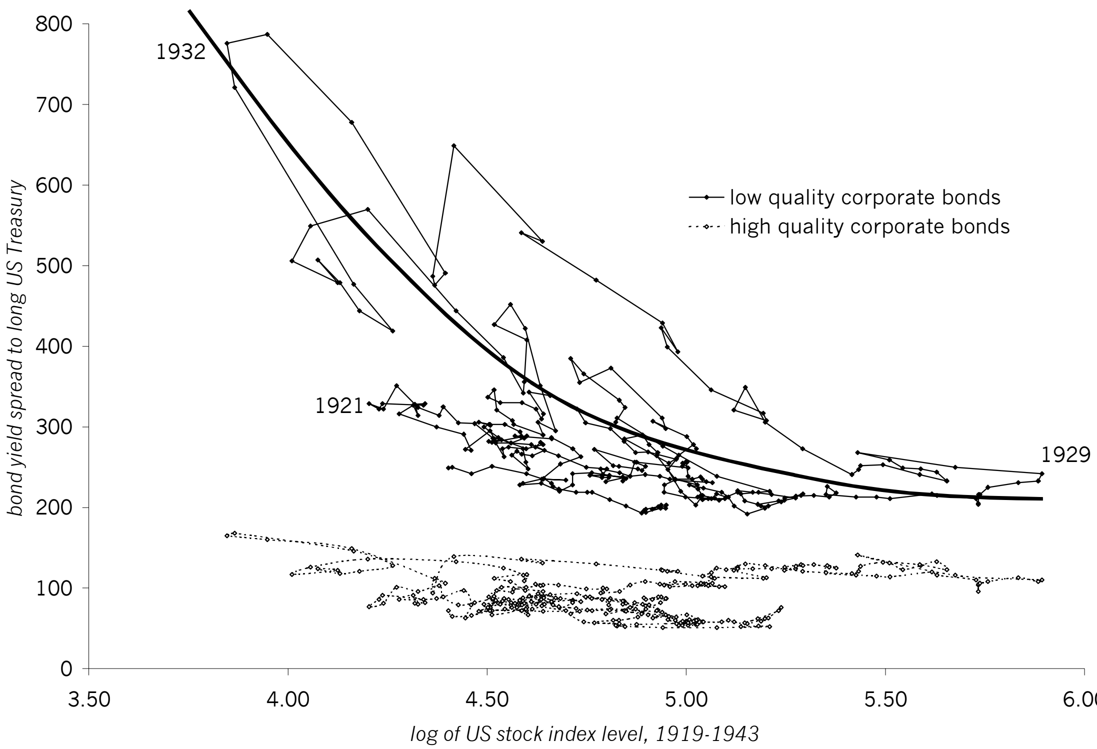

Exhibit 2c ■ US railroad stocks and railroad bond spreads, 1857-1929

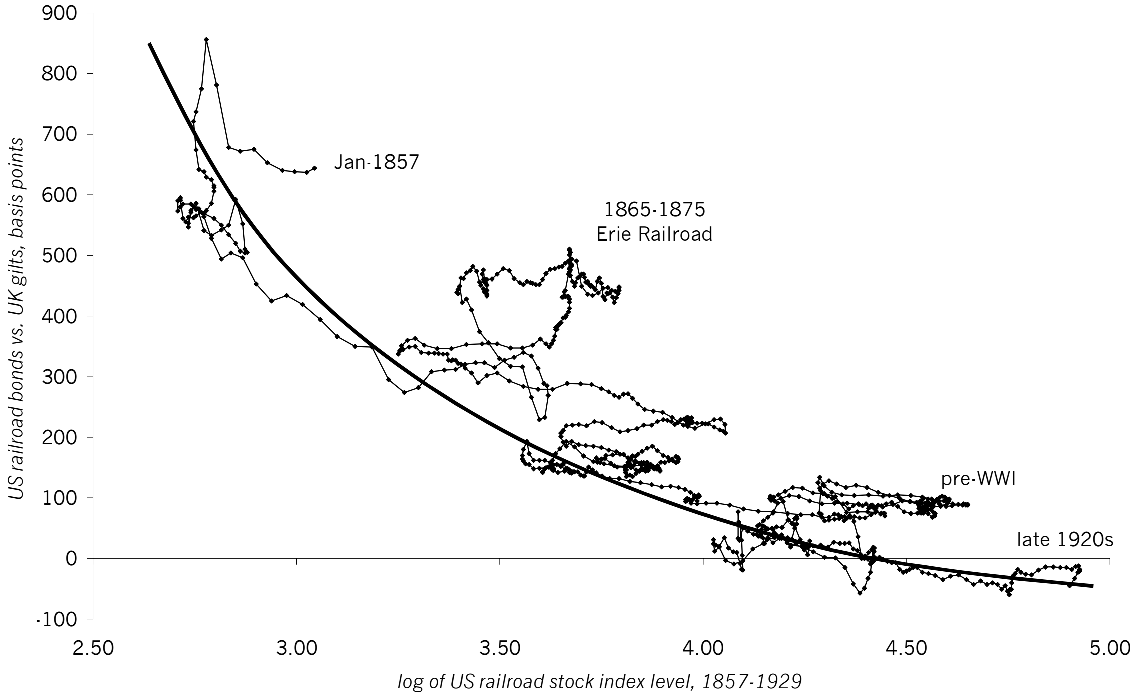

Exhibit 3a ■ Nextel: stock price and distance to default

Exhibit 3b ■ Nextel: volatility of enterprise value

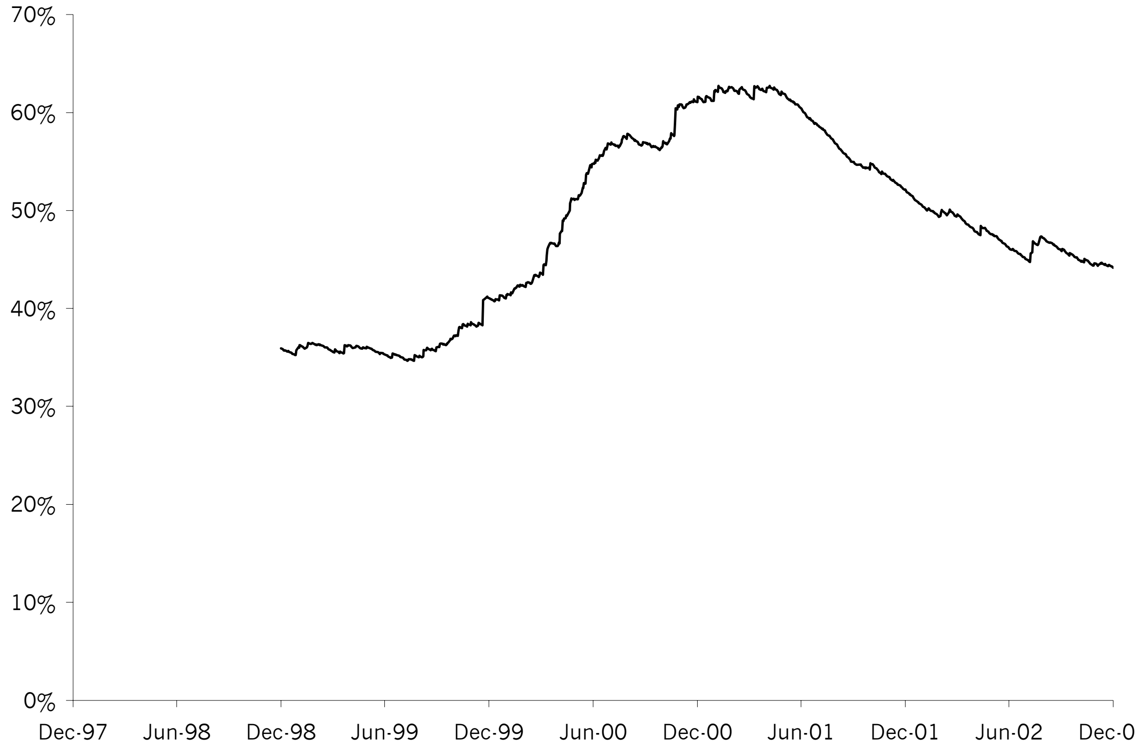

Exhibit 3c ■ Nextel: stock price and credit risk measure

Exhibit 3d ■ Nextel: credit risk measure and bond spread

Exhibit 3e ■ Nextel: bond spread vs. credit risk measure

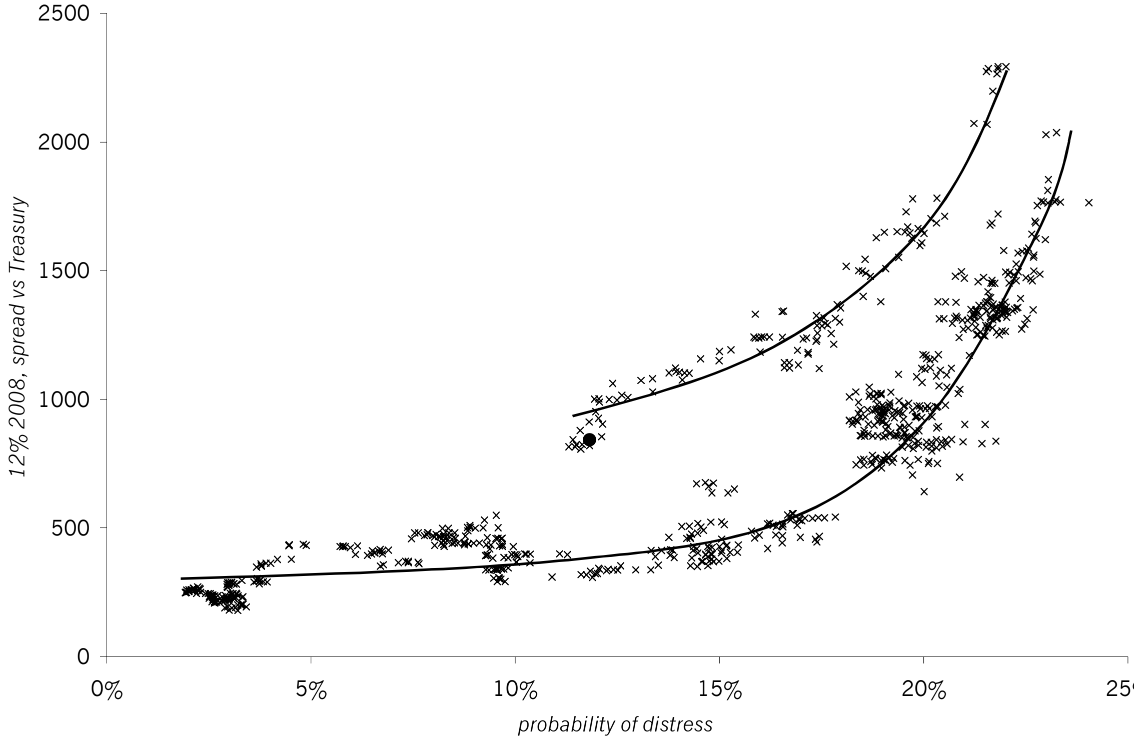

Exhibit 4a ■ Ford: credit risk measure, bond spread and rating actions

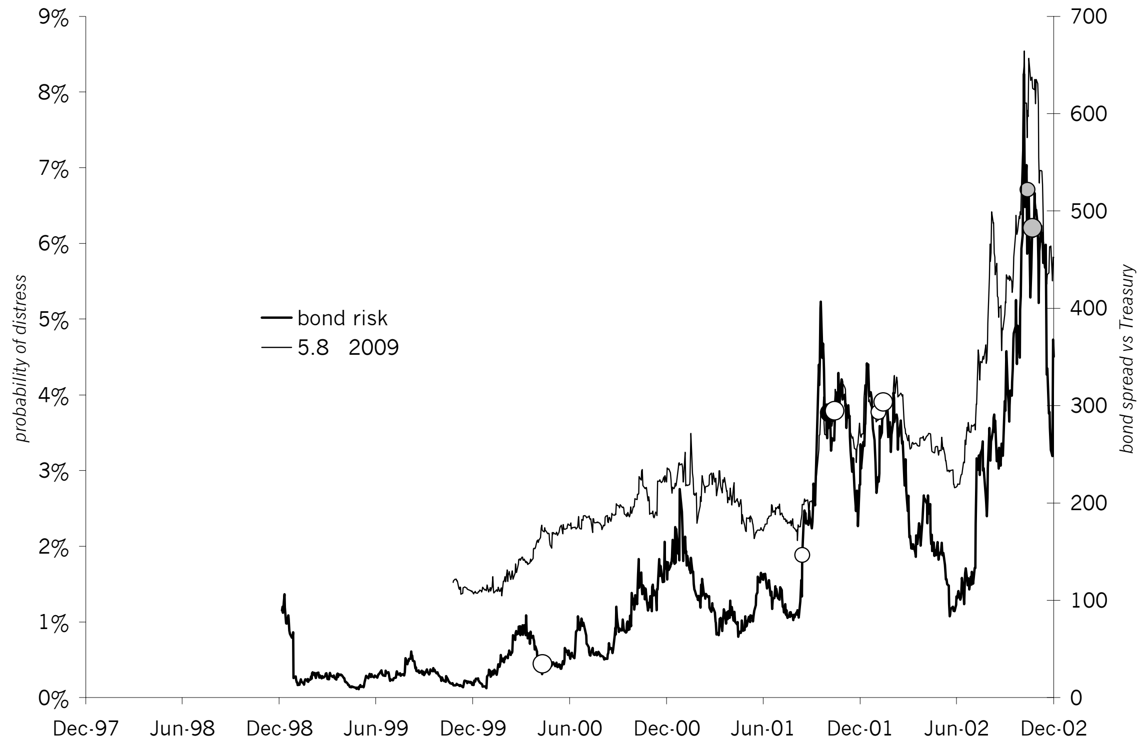

Exhibit 4b ■ Ford: bond spread vs. credit risk measure

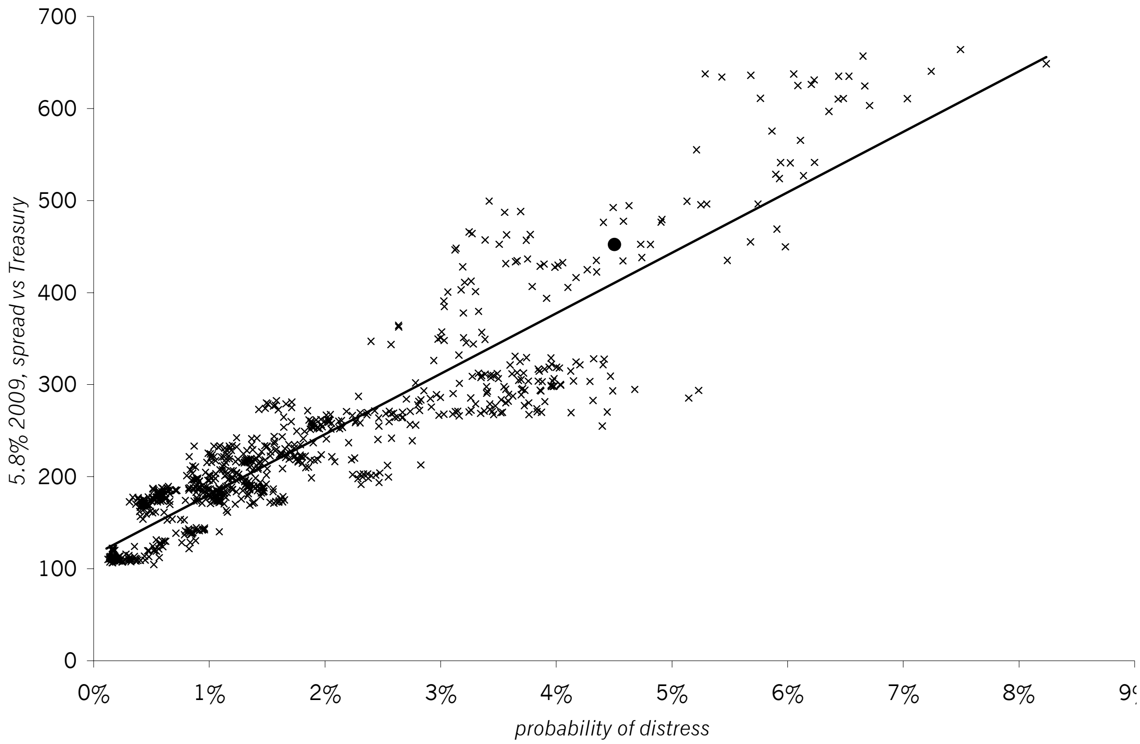

Exhibit 4c ■ Ford: stock price and critical range

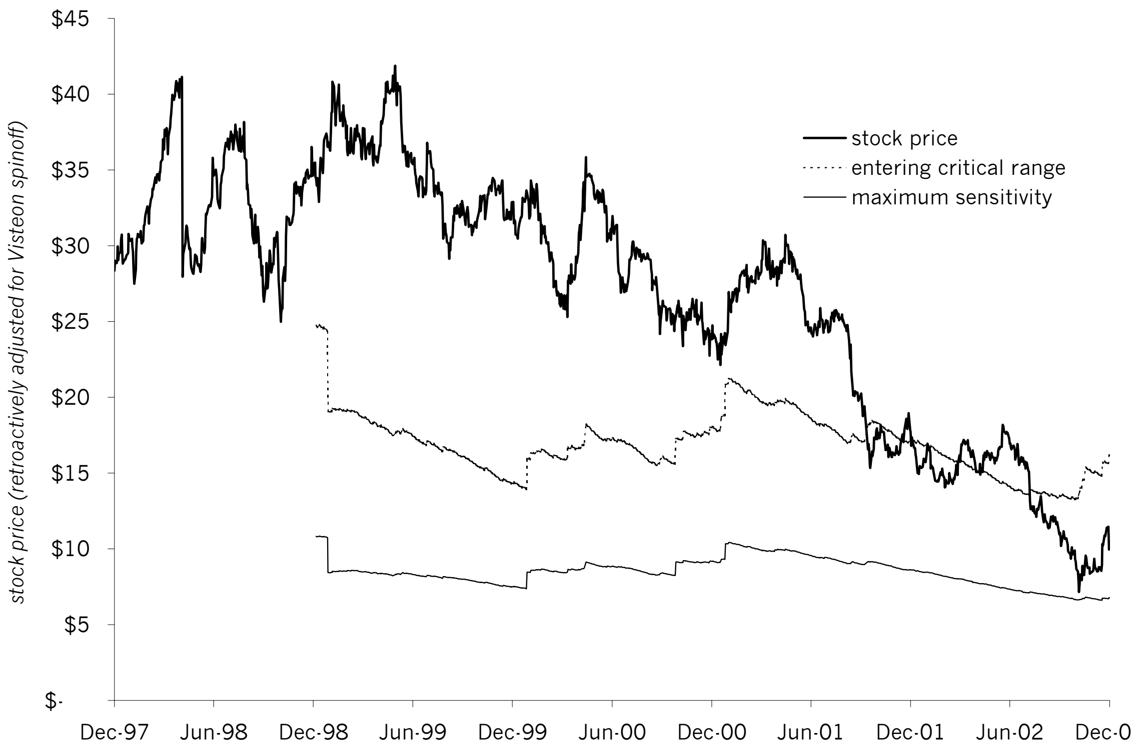

Exhibit 5a ■ Sprint: bond spread vs. credit risk measure

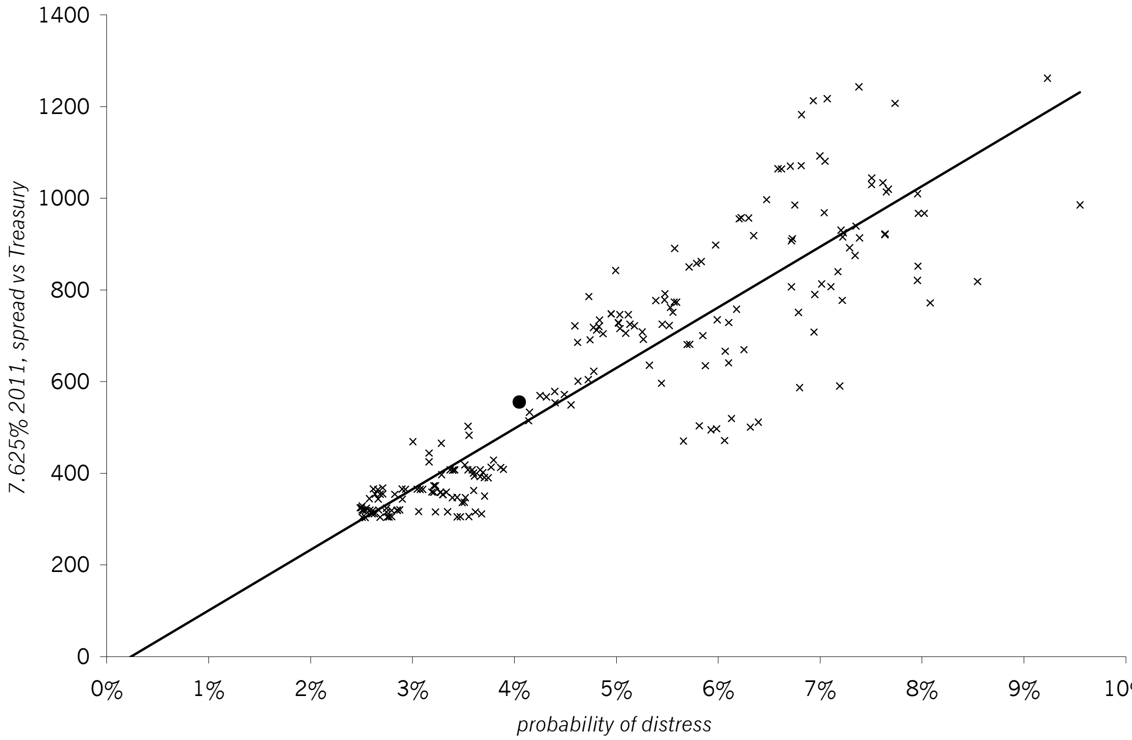

Exhibit 5b ■ Sprint: credit risk measure, bond spread and credit default swap spread

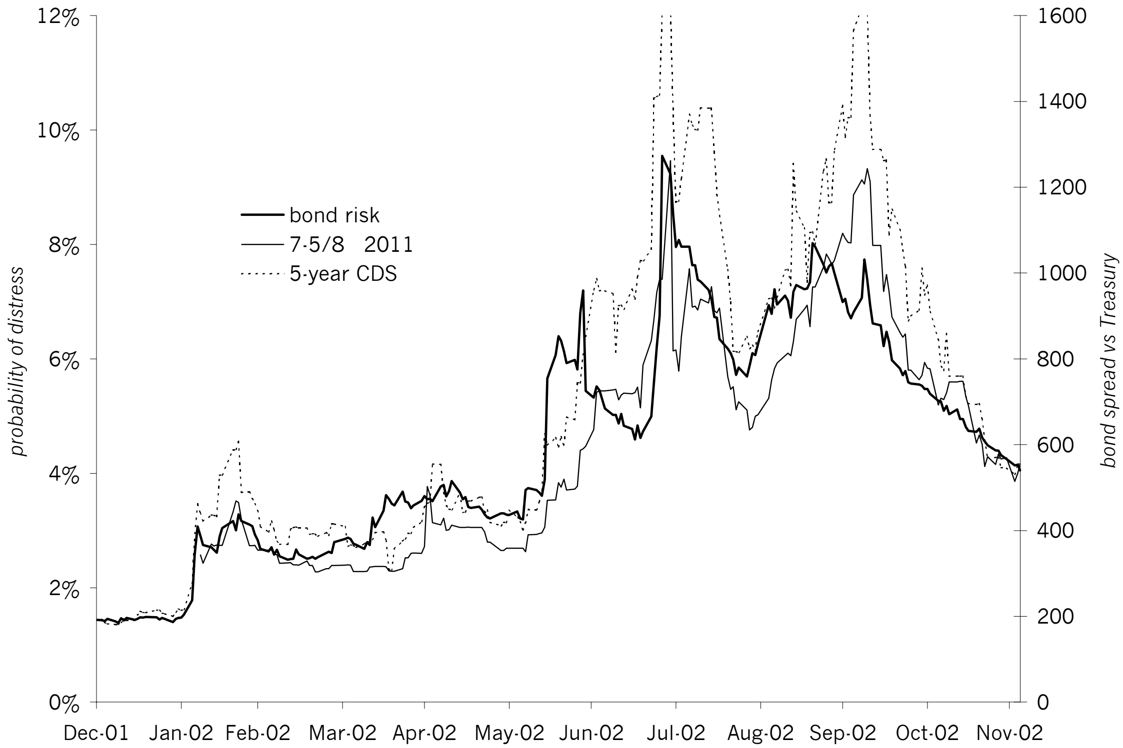

Exhibit 6a ■ Comparative equity return dynamics: static loan pricing

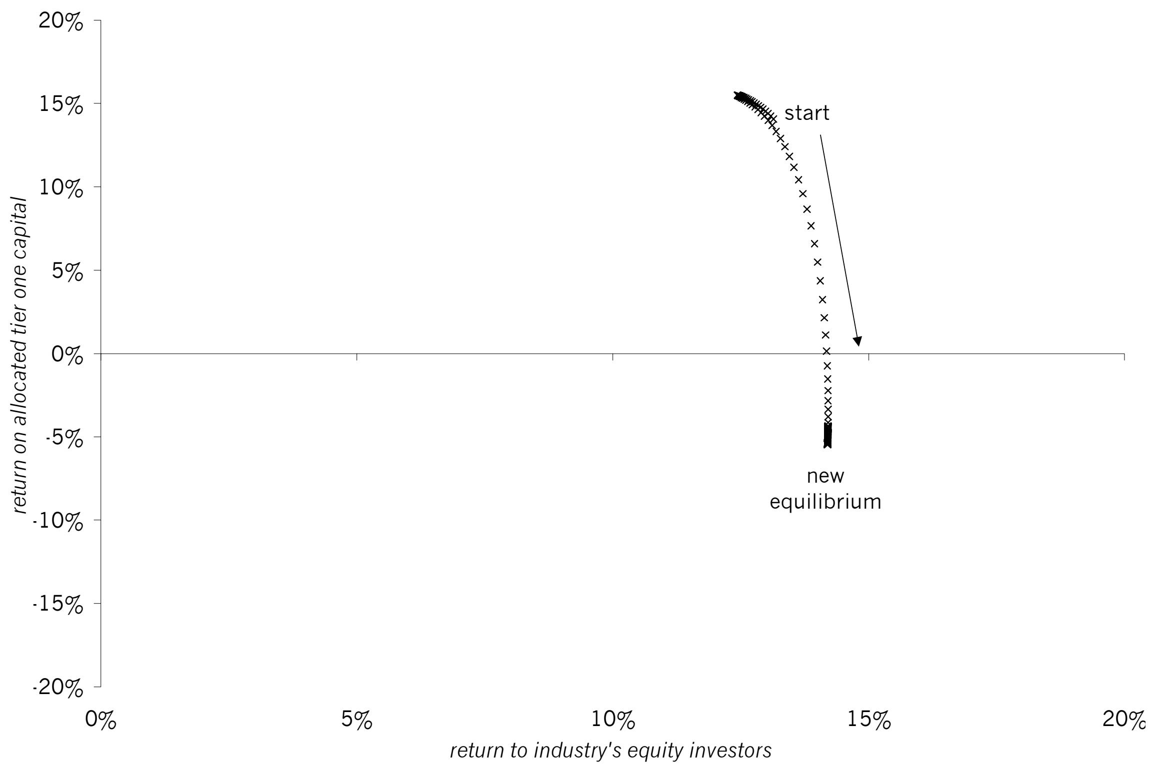

Exhibit 6b ■ Comparative equity return dynamics: risk based loan pricing